1. The yield to maturity of a $1,000 bond with a 7% coupon rate, semi-annual coupons and two years to maturity is 7.6% APR, compounded semi-annually. What is its price?

The cash flows from the bond are provided in the table below.

<table>
<thead>
<tr>
<th>Cash Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>6 months</td>
</tr>
<tr>
<td>1 year</td>
</tr>
<tr>
<td>1.5 years</td>
</tr>
<tr>
<td>2 years</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>$35</td>
</tr>
<tr>
<td>$35</td>
</tr>
<tr>
<td>$35</td>
</tr>
<tr>
<td>$1,000 + $35</td>
</tr>
</tbody>
</table>

Since the yield-to-maturity is an APR, compounded semi-annually, the appropriate discount rate for each cash flow is \( \frac{7.6\%}{2} = 3.8\% \). The price of the bond is the sum of the present values of its cash flows:

\[
PV = \frac{35}{1.038} + \frac{35}{(1.038)^2} + \frac{35}{(1.038)^3} + \frac{1,035}{(1.038)^4}
= $989.06
\]

2. You are analyzing bond quotes to evaluate an investment strategy for your firm. Unfortunately, some entries in the table are missing. You know that the face value is $1,000 and that coupon payments occur annually, the first one in a year from now. Complete all the necessary information below.

<table>
<thead>
<tr>
<th>Company</th>
<th>Bond Price</th>
<th>Coupon Rate</th>
<th>Maturity</th>
<th>YTM</th>
<th>Current Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1,034.96</td>
<td>5.625%</td>
<td>3 years</td>
<td>4.357%</td>
<td>5.435%</td>
</tr>
<tr>
<td>B</td>
<td>$870.00</td>
<td>7.5%</td>
<td>6 years</td>
<td>10.532%</td>
<td>8.621%</td>
</tr>
</tbody>
</table>
The bond price for Company A is found by using its current yield:

\[
\text{current yield} = \frac{\text{annual interest}}{\text{bond price}}
\]

\[
0.05435 = \frac{0.05625 \times \$1000}{\text{bond price}}
\]

\[\iff \]

\[
\text{bond price} = \frac{0.05625 \times \$1000}{0.05435} = \$1,034.96
\]

The coupon rate for Company B is found by first finding its annual interest:

\[
\text{current yield} = \frac{\text{annual interest}}{\text{bond price}}
\]

\[
0.08621 = \frac{\text{annual interest}}{\$870.00}
\]

\[\iff \]

\[
\text{annual interest} = 0.08621 \times \$870.00 = \$75.00
\]

Then, the coupon rate for Company B is:

\[
\text{coupon rate} = \frac{\text{annual interest}}{\text{face value}} = \frac{\$75.00}{\$1,000.00} = 7.5\%
\]

The yield-to-maturity (YTM) for Company A solves:

\[
\$1,304.96 = \frac{\$56.25}{(1 + YTM)} + \frac{\$56.25}{(1 + YTM)^2} + \frac{\$1,056.25}{(1 + YTM)^3},
\]

where the annual coupon is $56.25. Then, the YTM for Company A is 4.357%. Similarly, the yield-to-maturity for Company B solves:

\[
\$870.00 = \frac{\$75.00}{(1 + YTM)} + \frac{\$75.00}{(1 + YTM)^2} + \frac{\$75.00}{(1 + YTM)^3} + \frac{\$75.00}{(1 + YTM)^4} + \frac{\$75.00}{(1 + YTM)^5} + \frac{\$1,075.00}{(1 + YTM)^6},
\]

where the annual coupon is $75.00. Then, the YTM for Company B is 10.532%.

3. What is the price of a 1-year 3% coupon bond with semi-annual coupons when the 6-month interest rate is 2% and the 1-year interest rate is 2.6%? The par value of the bond is $1,000. Note: The 6-month and 1-year rate are quoted as APRs, compounded semi-annually. The cash flows from the bond are provided in the table below. Each cash flow must be valued using the appropriate discount rate. The 6-month rate is 2% APR, semi-annually compounded, which is a 6-month rate of \(\frac{2\%}{2} = 1\%\). The 1-year rate is 2.6% APR, compounded
semi-annually, which is a 6-month rate of $\frac{2.6\%}{2} = 1.3\%$. The price of the bond is the sum of the present values of its cash flows:

$$PV = \frac{15}{1.01} + \frac{1,015}{(1.013)^2}$$

$$= 1,003.97$$

**Note:** All of the discounting is done in terms of 6-month periods, so the first exponent is 1 for one 6-month period and the second exponent is 2 for two 6-month periods (1 year).

4. Consider a 2% coupon Treasury bond with exactly one year to maturity and a face value of $1,000. Coupons are paid semi-annually. The 1-year rate on the yield curve is quoted as 1% APR, compounded semi-annually, and the price for a 6-month STRIP is $99.50. What is the price of the bond?

A 2% coupon bond with semi-annual coupons pays a total of $20 (2% of $1,000) every year, broken into equal semi-annual payments of $10. The bond will also pay its par value ($1,000) at maturity. The cash flows from the bond are provided in the table below. The 6-month rate for the first 6 months can be found using the 6-month STRIP price: $\frac{100}{99.5} = 1.005025$. The 1-year rate is 1% APR, compounded semi-annually, which is a 6-month rate of $\frac{1\%}{2} = 0.5\%$.

The price of the bond is the sum of the present values of its cash flows:

$$PV = \frac{10}{1.005025} + \frac{1,010}{(1.005)^2}$$

$$= 1,009.93$$

5. Assume the following term structure of interest rates:

<table>
<thead>
<tr>
<th>Cash Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 months</td>
</tr>
<tr>
<td>$10</td>
</tr>
</tbody>
</table>

Interest rates are quoted as APRs with semi-annual compounding.

(a) What is the price of a 4% bond with semi-annual coupons that matures in exactly 1.5 years and has a par value of $1,000?

The cash flows from the bond are provided in the table below. The 6-month rate is 2% APR, semi-annually compounded, which is a 6-month rate of $\frac{2\%}{2} = 0.5\%$. The 1-year rate is 1.1% APR, compounded semi-annually, which is a 6-month rate of $\frac{1.1\%}{2} = 0.55\%$. 

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3
The 1.5-year rate is 1.4% APR, compounded semi-annually, which is a 6-month rate of \( \frac{1.4\%}{2} = 0.7\% \). The price of the bond is the sum of the present values of its cash flows:

\[
P V = \frac{\$20}{1.005} + \frac{\$20}{(1.0055)^2} + \frac{\$1,020}{(1.007)^3} = \$1,038.56
\]

(b) What is the price of a 1.5-year $1,000 zero-coupon bond (a $1,000 STRIP)?
For a STRIP, there is only one cash flow of $1,000 in 1.5 year. The 1.5-year rate is 1.4% APR, compounded semi-annually, which is a 6-month rate of \( \frac{1.4\%}{2} = 0.7\% \). The price of the bond is:

\[
P V = \frac{\$1000}{(1.007)^3} = \$979.29
\]

(c) Why does one sell at a premium and the other at a discount?
The bond sells at a premium because its coupon rate is greater than the market rates. The STRIP sells at a discount because it offers no coupons. It must sell at a discount to par value; otherwise it would have no return or a negative return (if it sold at a premium).

(d) What is the current yield of the bond? What is the current yield of the STRIP?
The current yield of the bond is:

\[
\text{current yield} = \frac{\text{annual interest}}{\text{bond price}} = \frac{\$40}{\$1,038.56} = 0.0385
\]

The current yield of the STRIP is 0%, since it does not pay interest.

(e) Which would have a higher yield-to-maturity: the bond or the STRIP?
The STRIP must have a higher yield-to-maturity. The STRIP’s yield-to-maturity is 1.4%, since this is the rate when the STRIP matures. The bond’s yield-to-maturity is a weighted average of 1%, 1.1% and 1.4%, so it will be close to, but slightly less than 1.4%.

6. You notice that the yield-to-maturity (YTM) of some zero-coupon Treasury bonds are:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>3 months</th>
<th>6 months</th>
<th>1 year</th>
<th>1.5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>1.0%</td>
<td>1.5%</td>
<td>1.8%</td>
<td>2%</td>
</tr>
</tbody>
</table>

All of the YTMs are quoted as APRs with semi-annual compounding.
(a) Based on the YTMs above, what is the price of a 1-year zero-coupon corporate bond with a credit spread of 80 basis points and par value of $1,000?

The cash flow for a 1-year zero-coupon is $1,000 in 1 year. The discount rate for this bond is the YTM on a 1-year Treasury bond plus the credit spread: $0.018 + 0.008 = 0.026$. Then, the price of the bond is the present value of its cash flow:

$$PV = \frac{\$1000}{(1 + 0.026)^2} = \$974.50$$

(b) Would the YTM of a 1.5-year 2% coupon Treasury bond be more than, less than, or equal to 2%?

The bond’s YTM would be less than 2%. The YTM of a 1.5-year zero-coupon bond is 2%, since it pays no coupons. The YTM of a 2% coupon bond is a weighted average of the 6 month, 1 year and 1.5 year rates. Since the 6 month and 1 year rates are less than 2%, the average must be less than 2%.

(c) Would the YTM of a 1.5-year 10% coupon bond be more than, less than, or equal to the YTM of a 1.5-year 5% coupon bond? Explain.

The YTM of a 1.5-year 10% coupon bond would be less than the YTM of a 1.5-year 5% coupon bond. This is because the coupon payments for the 10% bond are twice those of the coupon payments of the 5% bond. Then, in the weighted average in the YTM calculation, the 10% bond is putting relatively more weight on the earlier rates, which are the lower rates. So, the weighted average for the 10% bond must be lower than the weighted average for the 5% bond.

7. A corporate bond matures in 3 years, pays annual coupons, has a coupon rate of 4%, has a BBB credit rating and has a face value of $1,000. A 3-year Treasury bond that makes payments on the same days as the corporate bond has a yield to maturity of 2%. The credit spread on BBB bonds is 600 basis points. Additionally, the credit spread on AA bonds is 300 basis points.

(a) How much is each coupon payment on the corporate bond?

Since the coupon rate is 4% and coupons are paid annually, then each annual coupon payment is $1,000 * 0.04 = $40, assuming par value of $1,000.

(b) At what rate should you discount the corporate bond?

The BBB bond has a credit spread of 600 basis points. This is the spread over the Treasury bond. Then, the discount rate for the bond is 0.02 + 0.06 = 0.08.

(c) What is the price of the 3-year corporate bond?

The cash flows from the bond are provided in the table below:

<table>
<thead>
<tr>
<th>Cash Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
</tr>
<tr>
<td>$40</td>
</tr>
</tbody>
</table>

Using the 8% discount rate from above, the price of the bond is the sum of the present values of its cash flows:

$$PV = \frac{\$40}{(1 + 0.08)} + \frac{\$40}{(1 + 0.08)^2} + \frac{\$1000 + \$40}{(1 + 0.08)^3} = \$896.92$$
(d) Suppose the bond is upgraded to AA. What should the new price be?
When the bond is upgraded, we need to adjust the discount rate. AA bonds have a
credit spread of 300 basis points. Therefore, the discount rate for the upgraded bond is
$0.02 + 0.03 = 0.05$. Using the 5% discount rate and the same cash flow as above, the
price of the bond is the sum of the present values of its cash flows:

$$ PV = \frac{40}{1 + 0.05} + \frac{40}{(1 + 0.05)^2} + \frac{1000 + 40}{(1 + 0.05)^3} $$

$$ = 972.77 $$

8. Suppose that you purchased a bond a year ago and, over the past year, the market interest
rate increases. What will happen to the price of the bond? Explain the economic intuition
for the change in the bond’s price.

The price of the bond will decrease. Since the discount rate increases, then the price must
decrease, based on the formula. Economically, interest rates represent the opportunity cost
of capital. When rates increase and other investment opportunities become more attractive,
this one becomes relatively less attractive. Then, its price must decrease so investors are
willing to buy it.